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Technical Comment

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the Journal of Guidance, Control, and Dynamics *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Analytical Solution for Dynamic Analysis of a Flexible L-Shaped Structure"

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RECENTLY in this journal an interesting Engineering Note was published in which analytical solutions for an L-shaped beam, consisting of two elastic domains modeled as Bernoulli–Euler beams, were developed.[1] Because the ω^2 values were remarkably high in comparison with the physical parameters selected (representing a soft system), the present author was motivated to reexamine the derivations there. During these efforts some errors, which change the results obtained substantially, were detected.

First, there are some printing errors, which are as follows. A_{31}, A_{32}, and A_{33} should be A_{33}, A_{34}, and A_{35}, respectively. Second, the signs of A_{53}, A_{54}, and A_{55} have to be reversed, i.e., they should be positive. And finally and more seriously, in the element A_{54}, the term $-2/\gamma_2^2$ is forgotten in the bracket. This is an error that can occur easily. In the difference of the integrals

$$\int_0^{L_2} x_2 \cos\gamma_2 x_2 \, \mathrm{d}x_2, \qquad \int_0^{L_2} x_2 \cosh\gamma_2 x_2 \, \mathrm{d}x_2$$

the constant terms $-1/\gamma_2^2$ and $+1/\gamma_2^2$ do not eliminate each other but are summed.

For the sake of completeness, the corrected elements are

$$A_{33} = -\cos\gamma_2 l_2, \quad A_{34} = \sin\gamma_2 l_2 - \sinh\gamma_2 l_2, \quad A_{35} = \cosh\gamma_2 l_2$$

$$A_{53} = \rho_2\omega^2\left[\left(1/\gamma_2^2\right)\sin\gamma_2 l_2 - (l_2/\gamma_2)\cos\gamma_2 l_2\right]$$

$$A_{54} = \rho_2\omega^2\Big[\left(1/\gamma_2^2\right)\cos\gamma_2 l_2 + (l_2/\gamma_2)\sin\gamma_2 l_2 - (l_2/\gamma_2)\sinh\gamma_2 l_2 + \left(1/\gamma_2^2\right)\cosh\gamma_2 l_2 - 2/\gamma_2^2\Big]$$

$$A_{55} = \rho_2\omega^2\left[(l_2/\gamma_2)\cosh\gamma_2 l_2 - \left(1/\gamma_2^2\right)\sinh\gamma_2 l_2\right]$$

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Table 1 Corrected numerical results

Mode no.	ω^2, rad^2/s^2
1	0.050503
2	0.630446
3	7.189738
4	33.310309
5	65.235284
6	215.106384
7	344.298068

Hence, the frequency equation (14) (Ref. 1) is entirely erroneous and has to be replaced by the following longer expression, which is obtained essentially by the expansion of the determinant of the matrix A with the help of MATHEMATICA:

$$\begin{aligned}
&4E_1^2 I_1^2\gamma_1^4\gamma_2^3[1 + \cos\gamma_2 l_2\cosh\gamma_2 l_2 + \cos\gamma_1 l_1\cosh\gamma_1 l_1] \\
&+ \left(E_1^2 I_1^2\gamma_1^4\gamma_2^3 - 2E_1 I_1\gamma_1^3\rho_2\omega^2\right)\cosh\gamma_n\cos\gamma_n \\
&+ \left(E_1^2 I_1^2\gamma_1^4\gamma_2^3 + 2E_1 I_1\gamma_1^3\rho_2\omega^2\right)\cosh\gamma_p\cos\gamma_p \\
&- 4E_1 I_1\gamma_1\gamma_2^3 m_2\omega^2[\cosh\gamma_1 l_1\sin\gamma_1 l_1 - \cos\gamma_1 l_1\sinh\gamma_1 l_1] \\
&- 4m_2\rho_2\omega^4[\cos\gamma_2 l_2\sinh\gamma_2 l_2 - \sin\gamma_2 l_2\cosh\gamma_2 l_2] \\
&- \left(E_1 I_1\gamma_1\gamma_2^3 m_2\omega^2 - m_2\rho_2\omega^4\right)[\cosh\gamma_n\sin\gamma_n \\
&+ \cosh\gamma_p\sin\gamma_n - \sinh\gamma_n\cos\gamma_n - \sinh\gamma_n\cos\gamma_p] \\
&- \left(E_1 I_1\gamma_1\gamma_2^3 m_2\omega^2 + m_2\rho_2\omega^4\right)[\cosh\gamma_n\sin\gamma_p \\
&+ \cosh\gamma_p\sin\gamma_p - \sinh\gamma_p\cos\gamma_n - \sinh\gamma_p\cos\gamma_p] \\
&+ E_1^2 I_1^2\gamma_1^4\gamma_2^3[\cosh\gamma_n\cos\gamma_p + \cosh\gamma_p\cos\gamma_n] \\
&- 2E_1 I_1\gamma_1^3\rho_2\omega^2[\sinh\gamma_n\sin\gamma_p - \sinh\gamma_p\sin\gamma_n] = 0
\end{aligned}$$

where the abbreviations

$$\gamma_p = \gamma_1 l_1 + \gamma_2 l_2, \qquad \gamma_n = \gamma_1 l_1 - \gamma_2 l_2$$

are introduced.

As a natural result of the preceding arguments, the squares of the eigenfrequencies listed in Table 2 of Ref. 1 are also incorrect and have to be corrected as given in Table 1 here.

Reference

[1]Bang, H., "Analytical Solution for Dynamic Analysis of a Flexible L-Shaped Structure," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 1, 1996, pp. 248–250.